Math 3353, Spring 2017  
Due February 17  

Homework 4 – The Matrix Equation $A\vec{x} = \vec{b}$, Solution Sets and Linear Independence

1. Consider the matrix $A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 0 & -1 & -1 & -1 \\ -2 & 2 & 2 & 3 \end{bmatrix}$

(a) How many rows of $A$ contain a pivot position?

(b) Do the columns of $A$ span $\mathbb{R}^3$?

(c) Does the equation $A\vec{x} = \vec{b}$ have a solution for every $\vec{b} \in \mathbb{R}^3$?

(d) Would the equation $A\vec{x} = \vec{0}$ have a nontrivial solution?

(e) Are the columns of $A$ linearly independent?

2. Let $A \in \mathbb{R}^{m\times n}$, let $\vec{v}$ and $\vec{w}$ be vectors in $\mathbb{R}^n$ such that $A\vec{v} = \vec{0}$ and $A\vec{w} = \vec{0}$, let $\vec{x} \in \mathbb{R}^n$ such that $A\vec{x} = \vec{b}$, and let $c$ and $d$ be arbitrary constants. Prove that the product $A(c\vec{v} + d\vec{w} + \vec{x})$ equals $\vec{b}$, using only the properties of matrix-vector products. Show each step of your proof, and write a short justification for each step.

3. For which value(s) of $h$ is the following set of vectors linearly dependent?

$$\left\{ \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \right\}$$

4. **MATLAB:** Using rref, determine if the columns of the following matrices span $\mathbb{R}^4$:

$$A = \begin{bmatrix} -7 & 9 & 2 & 15 \\ 6 & 10 & -2 & 7 \\ 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 5 & -2 & -10 \\ -2 & 1 & -3 & 0 & 5 \\ 1 & 0 & 2 & 0 & -1 \\ -1 & 1 & -1 & 1 & 6 \end{bmatrix},$$

$$C = \begin{bmatrix} -3 & 4 & -7 & 2 & 7 \\ 5 & 11 & -6 & -7 & 12 \\ 11 & 5 & 6 & -9 & -3 \\ -7 & -3 & -4 & 6 & -9 \end{bmatrix}, \quad D = \begin{bmatrix} -4 & 8 & 1 & 4 \\ -2 & 5 & -1 & 2 \\ 2 & -3 & 1 & -2 \\ -2 & 1 & 2 & 7 \end{bmatrix}.$$  

For each matrix, answer “Yes” or “No”, by writing your answer on the printed diary.