Math 3353, Spring 2017  
Due Monday April 17  

Homework 10 – Diagonalization and Complex Eigenvalues

1. Determine whether the following matrices are diagonalizable, are not diagonalizable, or that you have insufficient information to determine; justify each answer.
   (a) $A \in \mathbb{R}^{5 \times 5}$ has two distinct eigenvalues. One eigenspace is two-dimensional, the other is three-dimensional.
   (b) $A \in \mathbb{R}^{4 \times 4}$ has four distinct eigenvalues.
   (c) $A \in \mathbb{R}^{4 \times 4}$ has two distinct eigenvalues. One eigenspace is three-dimensional.
   (d) $A \in \mathbb{R}^{5 \times 5}$ has four distinct eigenvalues. Each eigenspace is one-dimensional.
   (e) $A \in \mathbb{R}^{8 \times 8}$ has five distinct eigenvalues. One eigenspace is three-dimensional.

2. Diagonalize the matrix $A = \begin{bmatrix} -4 & 0 & 2 & 0 \\ 0 & -2 & 0 & 4 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -6 \end{bmatrix}$.

3. Find the eigenvalues their corresponding eigenvectors for the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 4 \end{bmatrix}$.

4. **MATLAB:** In this problem you will experiment with random matrices to try to determine whether each of the following “properties” of eigenvalues is either True or False:
   (a) The product of the eigenvalues of a matrix $A$ is equal to the determinant of $A$.
   (b) The eigenvalues of the power of a matrix equal the power of the eigenvalues of the matrix, i.e. if $\lambda$ is an eigenvalue of $A$, then $\lambda^k$ is an eigenvalue $A^k$.
   (c) Similar matrices have the same eigenvalues, i.e. if $A = PBP^{-1}$ then $A$ and $B$ have the same eigenvalues.
   (d) Two matrices that are row equivalent have the same eigenvalues, e.g. if $B = \text{rref}(A)$ then $A$ and $B$ have the same eigenvalues.
   (e) The eigenvalues of a multiple of a matrix equal the same multiple of the eigenvalues of the matrix, i.e. if $B = cA$ and if $\lambda$ is an eigenvalue of $A$, then $c\lambda$ is an eigenvalue of $B$.
   (f) The eigenvalues of the sum of two matrices are the sum of the eigenvalues of the two matrices, i.e. if $\lambda$ is an eigenvalue of $A$ and $\gamma$ is an eigenvalue of $B$, then $(\lambda + \gamma)$ is an eigenvalue of $(A + B)$.

To draw conclusions about the above properties, I suggest that you use random $4 \times 4$ matrices; you can create these using MATLAB’s `rand` function, e.g.
I also suggest that you try each calculation at least twice with different matrices. Turn in a diary showing these experiments (6 questions, at least 2 experiments each, for a total of at least 12).

**Hints:**

- The MATLAB command to take the determinant of a square matrix is `det`, e.g.
  $$\text{>> det}(A)$$
  will compute the determinant of $A$.
- The MATLAB command to compute the product of entries in a vector is `prod`, e.g.
  $$\text{>> prod}(\text{eig}(B))$$
  will compute the product of the eigenvalues of $B$.
- The MATLAB command to exponentiate entries of a vector is `.^`, e.g.
  $$\text{>> (eig}(C))^.^3$$
  will compute the cube of each eigenvalue of $C$.
- You can create similar matrices $B$ and $A = PBP^{-1}$ with the commands
  $$\text{>> B = rand}(4,4); \quad P = \text{rand}(4,4); \quad A = P*B*\text{inv}(P);$$