Math 3353, Spring 2017

The following will be the instructions on your in-class exam (the number of questions may vary):

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Justify your answers algebraically whenever possible to ensure full credit.

- No calculators or other electronic devices are permitted without the explicit consent of the instructor.

- This exam is open book and open notes.

- Circle or otherwise indicate your final answers.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 10 problems and is worth 200 points. It is your responsibility to make sure that you have all of the pages!

- Good luck!
1. Compute \[
\begin{vmatrix}
1 & -2 & 5 & 2 \\
0 & 0 & 3 & 0 \\
2 & -6 & -7 & 5 \\
5 & 0 & 4 & 0 \\
\end{vmatrix}
\] using a method of your choosing.
2. If \[ \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7, \] compute \[ \begin{vmatrix} 2d & 2e & 2f \\ 2a & 2b & 2c \\ 2g & 2h & 2i \end{vmatrix}. \]
3. Use Cramer’s rule to solve the linear system $A\vec{x} = \vec{b}$ for only the solution component $x_2$,

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 4 \\ 3 \\ 4 \\ 0 \end{bmatrix}$$
4. Let $W$ be the set of vectors of the form
\[
\begin{bmatrix}
0 \\
2a + b \\
3b - a \\
4a - 7b
\end{bmatrix},
\]
where $a$ and $b \in \mathbb{R}$. If $W$ is a vector space find a set of vectors that spans $W$; otherwise prove that $W$ is not a vector space.
5. Consider \( A = \begin{bmatrix} 2 & -5 & -2 & 6 & 1 \\ -2 & 5 & 0 & 1 & 0 \end{bmatrix} \). Find \( p \) such that \( \text{Nul}(A) \) is a subspace of \( \mathbb{R}^p \). Find \( q \) such that \( \text{Col}(A) \) is a subspace of \( \mathbb{R}^q \). Write two nonzero vectors, \( \vec{x} \in \text{Nul}(A) \) and \( \vec{y} \in \text{Col}(A) \).
Let \( A = \begin{bmatrix} 2 & 4 & -4 & 6 & 0 & 2 \\ -3 & -6 & 6 & -9 & 1 & 2 \\ -1 & -2 & 2 & -3 & 3 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \). Find bases for \( \text{Nul}(A) \) and \( \text{Col}(A) \). What are the dimensions of these two subspaces?
7. Suppose that \((\vec{v}, \alpha)\) is an eigenpair of an invertible matrix \(A\) (hence \(\vec{v}\) is also an eigenvector of \(A^{-1}\)). Also suppose that \((\vec{v}, \beta)\) is an eigenpair of a matrix \(B\). Show that \(\vec{v}\) is an eigenvector of the matrix \(C = (3A^{-1} + B)\), and find its corresponding eigenvalue, \(\lambda\).
Assume that $A = QBQ^{-1}$, where $A = \begin{bmatrix} -15 & 21 & -7 \\ -12 & 17 & -5 \\ -8 & 12 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 2 & 0 \\ -1 & 2 & -1 \end{bmatrix}$.

Find the eigenvalues of $A$. 
9. The matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ has eigenvalue/eigenvector pairs $\lambda_1 = 5$, $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = -2$, $\vec{v}_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$. Diagonalize $A$ (i.e. write 3 matrices, $P$, $D$ and $P^{-1}$ such that $A = PDP^{-1}$).
10. The matrix $A = \begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$ has an eigenvector/eigenvalue pair $\lambda_1 = 3 - i$, $\vec{v}_1 = \begin{bmatrix} 5 \\ 2 + i \end{bmatrix}$.

(a) What is the other eigenpair?

(b) Find an invertible matrix $P$ and a rotation matrix $C$ such that $A = PCP^{-1}$. 
