1. Consider the following 3 data points:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>28</td>
<td>2</td>
<td>-8</td>
</tr>
</tbody>
</table>

Interpolate the data with a polynomial using a method of your choosing (tell me which method you use). Write the interpolating function (do not simplify). Use it to estimate \( f(0) \).

2. Suppose you wish to use \( n+1 \) points to interpolate the function \( \sin \left( \frac{x}{2} \right) \) on the interval \([0, 2]\).

Write a formula that \( n \) must satisfy in order to guarantee that the error in this interpolation is below \( 10^{-4} \), assuming

(a) the nodes are uniformly-spaced within \([0, 2]\), and

(b) the nodes are randomly placed within \([0, 2]\).

3. Using the 3 points \( f(x) \), \( f(x - h) \) and \( f(x + 3h) \) we want to derive an approximation

\[
f''(x) \approx w_1 f(x) + w_2 f(x - h) + w_3 f(x + 3h).
\]

(a) Write the linear system that you must solve to find the coefficients \( w_j \) (do not solve).

(b) If the solution to this system is \( w_1 = -\frac{2}{3h^2} \),  \( w_2 = \frac{1}{2h^2} \), \( w_3 = \frac{1}{6h^2} \), write the approximation formula for \( f''(x) \).

(c) What is the order of accuracy for this approximation (e.g. \( O(h^8) \))? 

4. Approximate the integral

\[
\int_{-1}^{1} (x^2 - 2x - 1) \, dx
\]

using (a) the midpoint rule, (b) the trapezoid rule, and (c) Simpson’s rule. Give the error in each of these approximations.
5. Suppose that you want to approximate the integral
\[ \int_{-1}^{1} \frac{dx}{2 + x} \]
using the composite Trapezoidal and composite Simpson rules over a uniform partition, \( P = \{x_0 < x_1 < \ldots < x_n\} \), \( T(f;P) \) and \( S(f;P) \). Use the asymptotic error formulas from class to write formulas for the minimum number of subintervals required for each method to guarantee an error below \( 10^{-8} \). [i.e. 2 formulas of the form “\( n > \ldots \)”]

6. Find the coefficients \( w_1 \) and \( w_2 \) that provide the best approximation of the integral
\[ \int_{-1}^{1} f(x) \, dx \approx w_1 f\left(-\frac{1}{2}\right) + w_2 f\left(\frac{1}{3}\right). \]
Write the resulting formula. What is its degree of precision?